Thompson Partition Monoids

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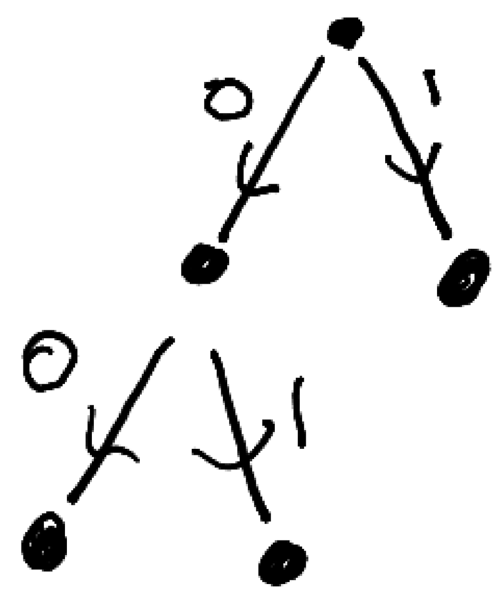
(joint with Naftoli Kolodny)

Outline

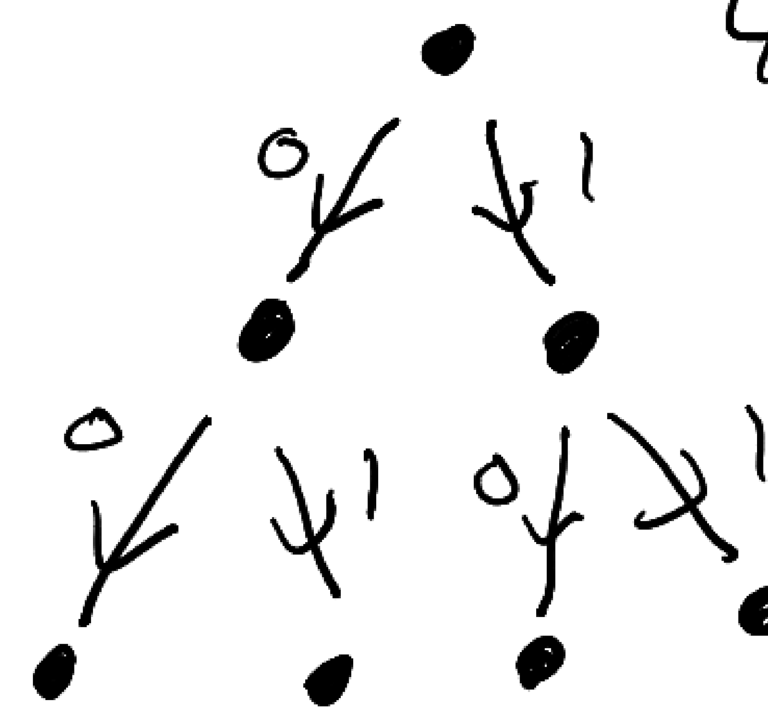
- Thompson Background
- Partition Monoid Background
- How to work in PV
- Jonsson-Tarski Algebra Viewpoint
- Questions

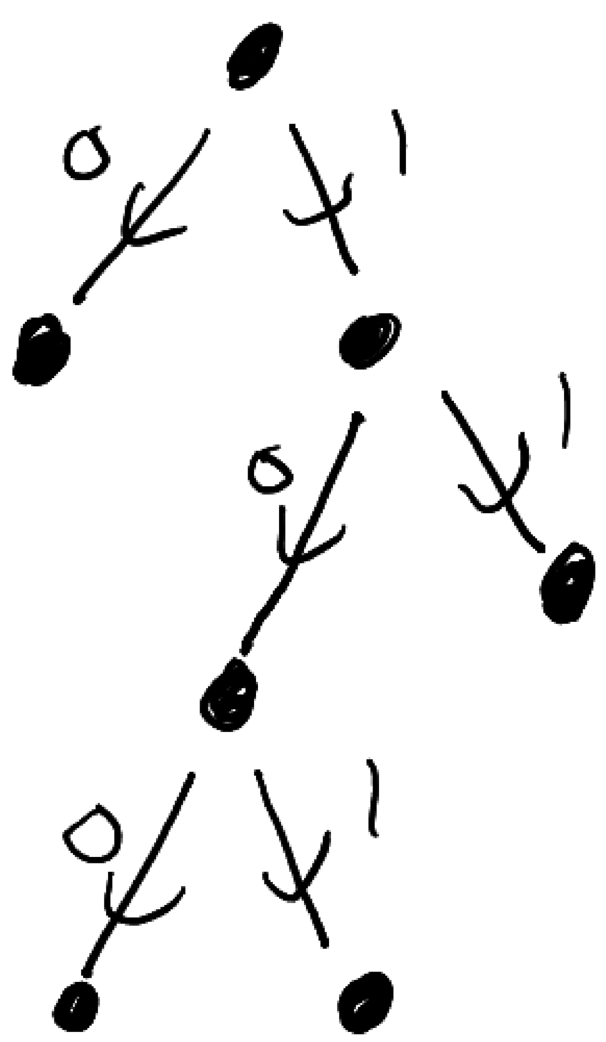
What is V (binary trees)?

3 leaves

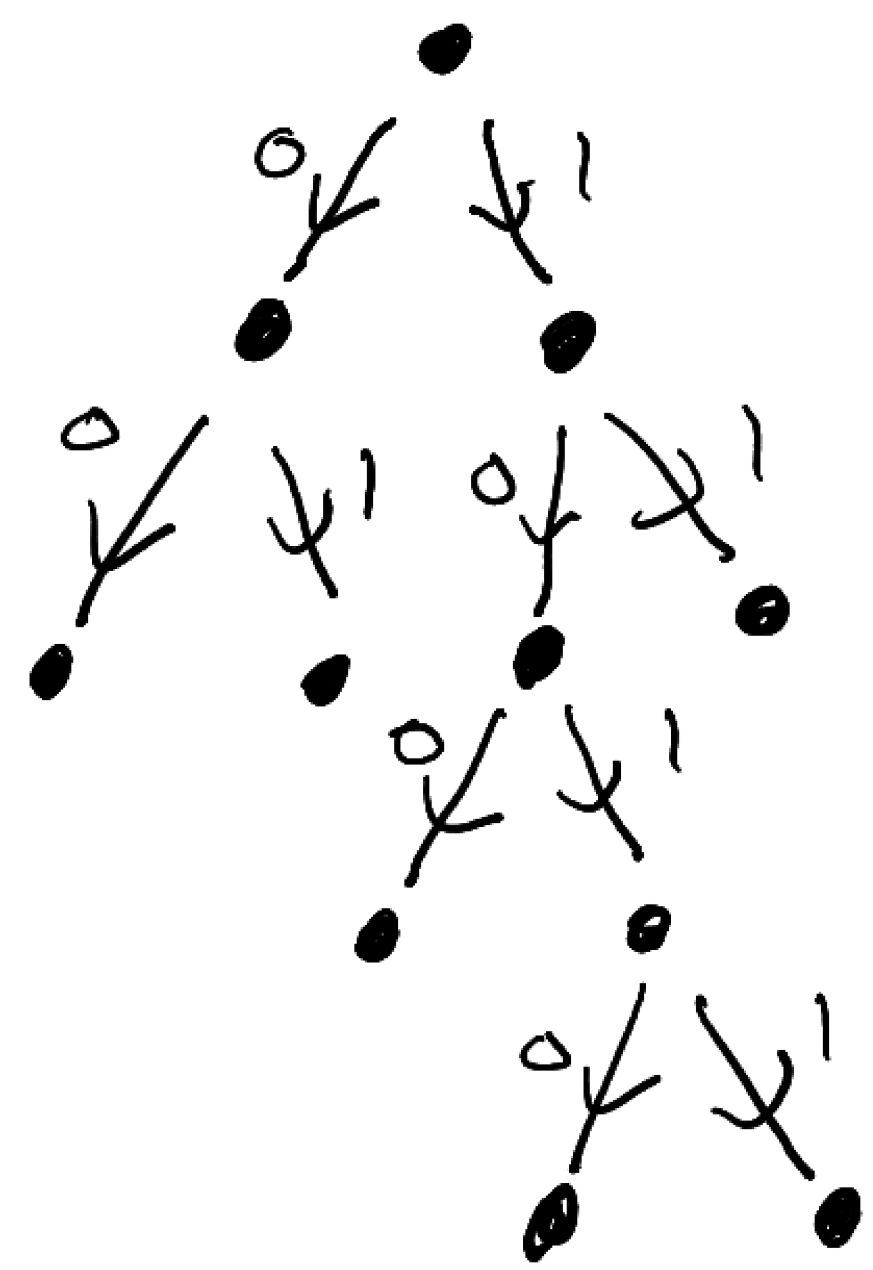


4 leaves

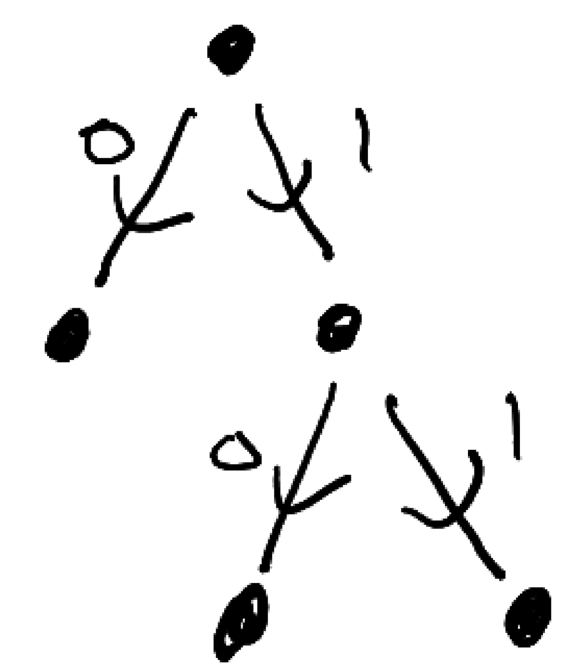




4 leaves

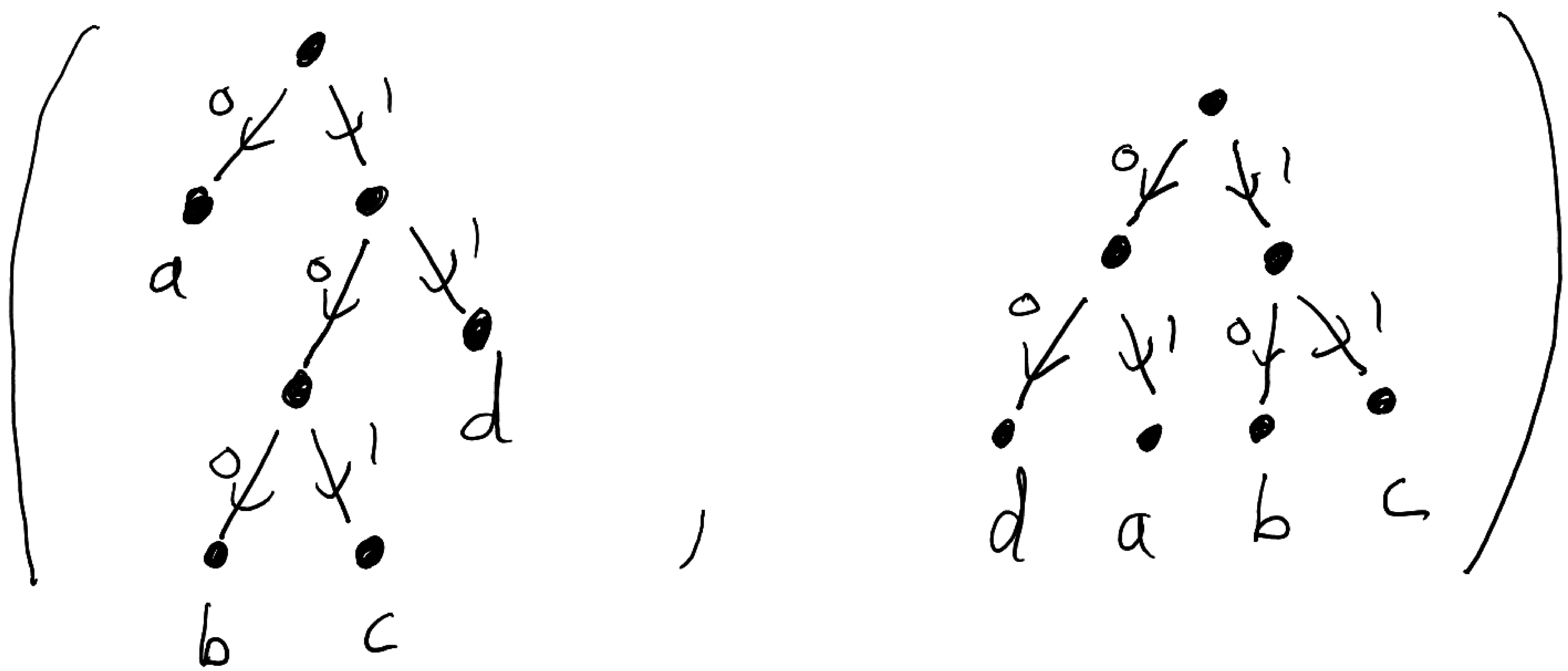


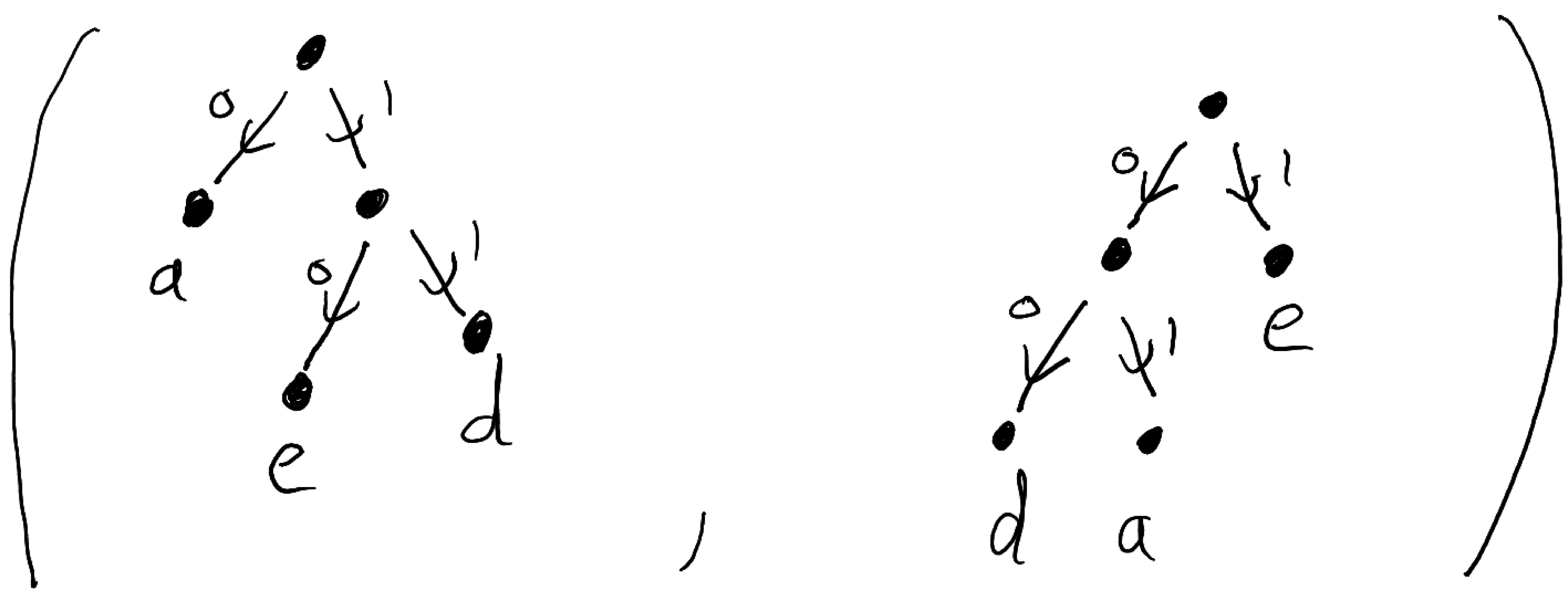
6 leaves



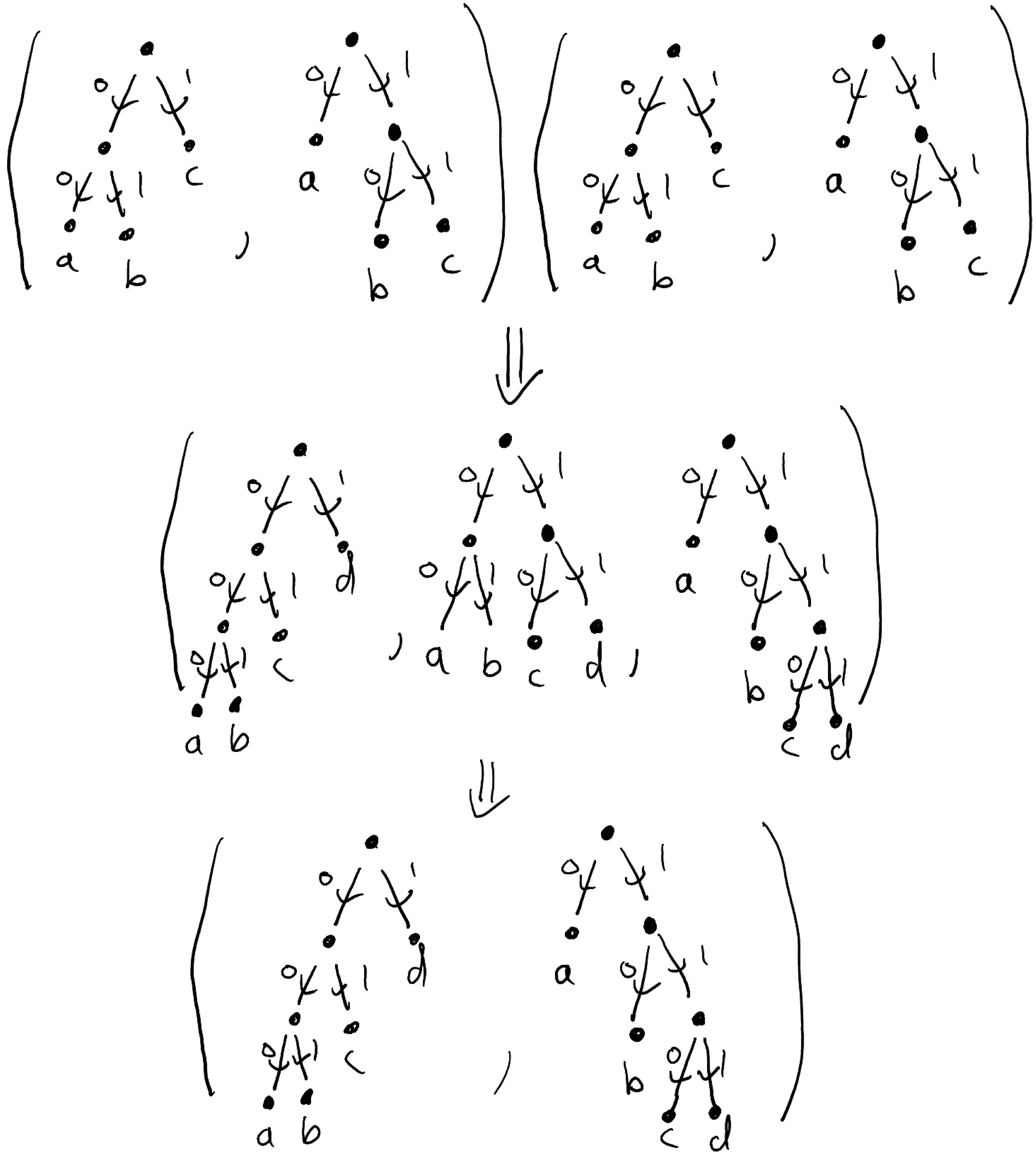
3 leaves

What is V (tree pairs)?

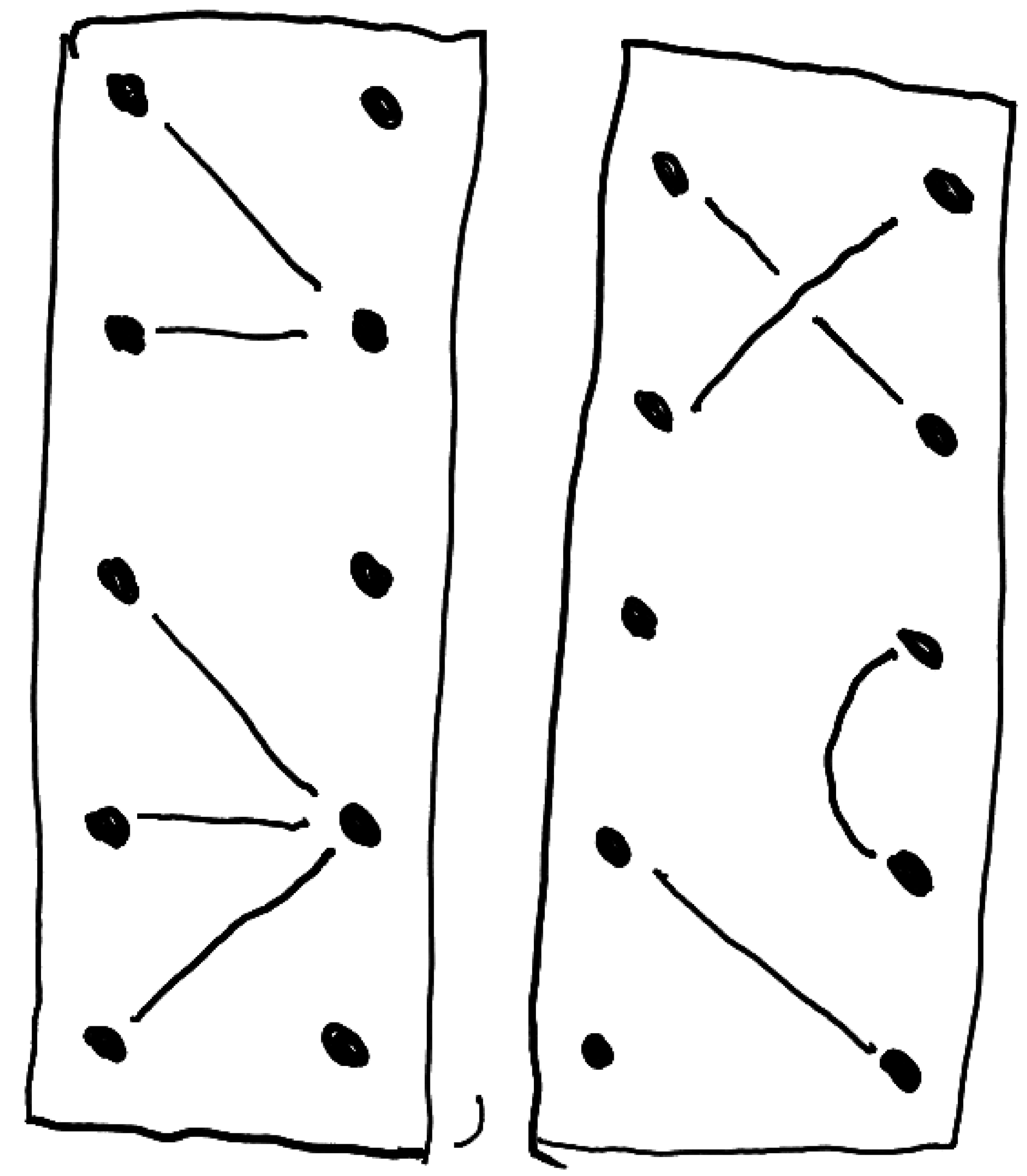




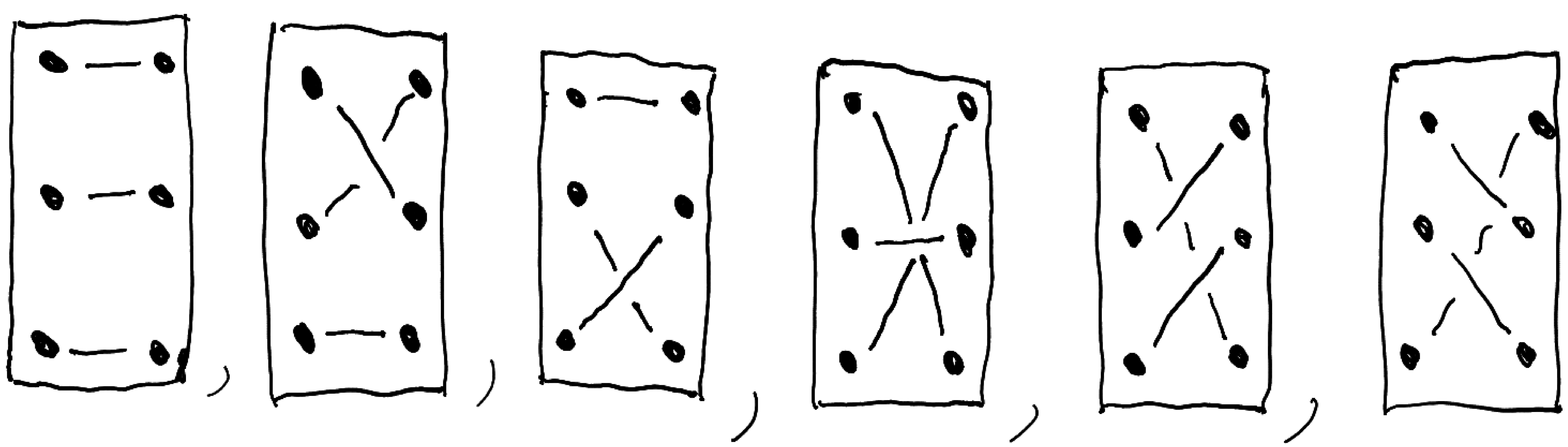
Multiplication in V



What are the Partition monoids P_n?



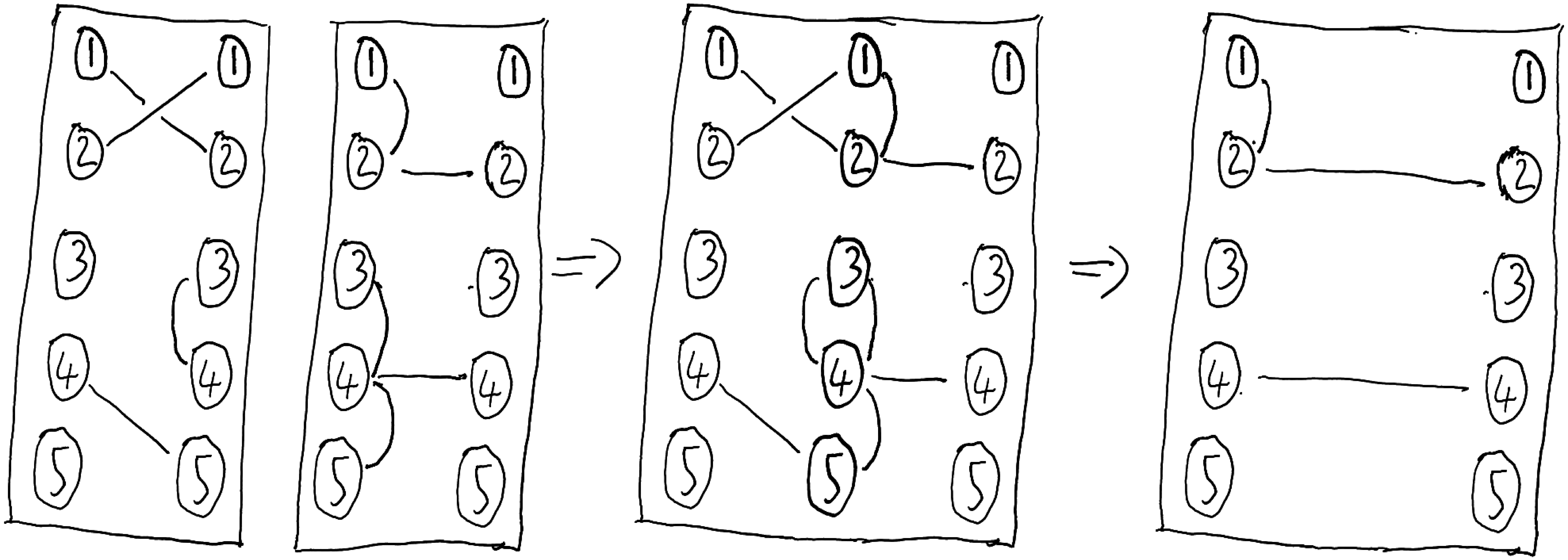
$\in P_5$

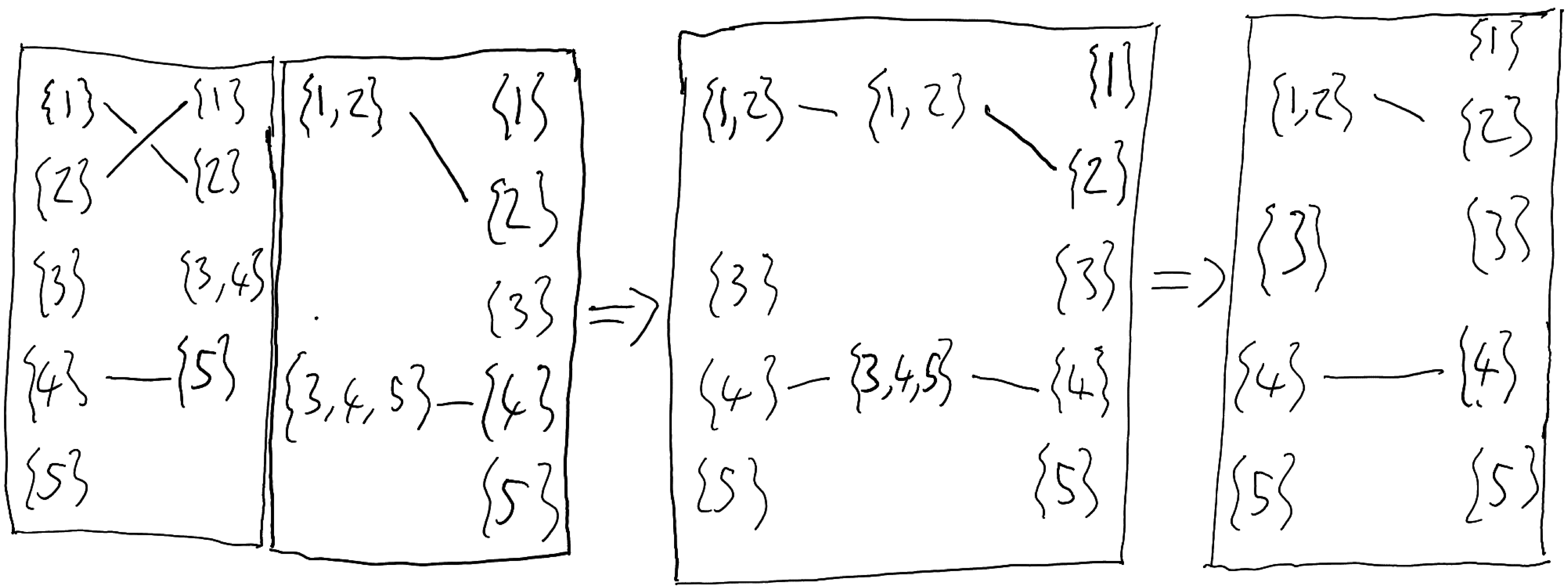


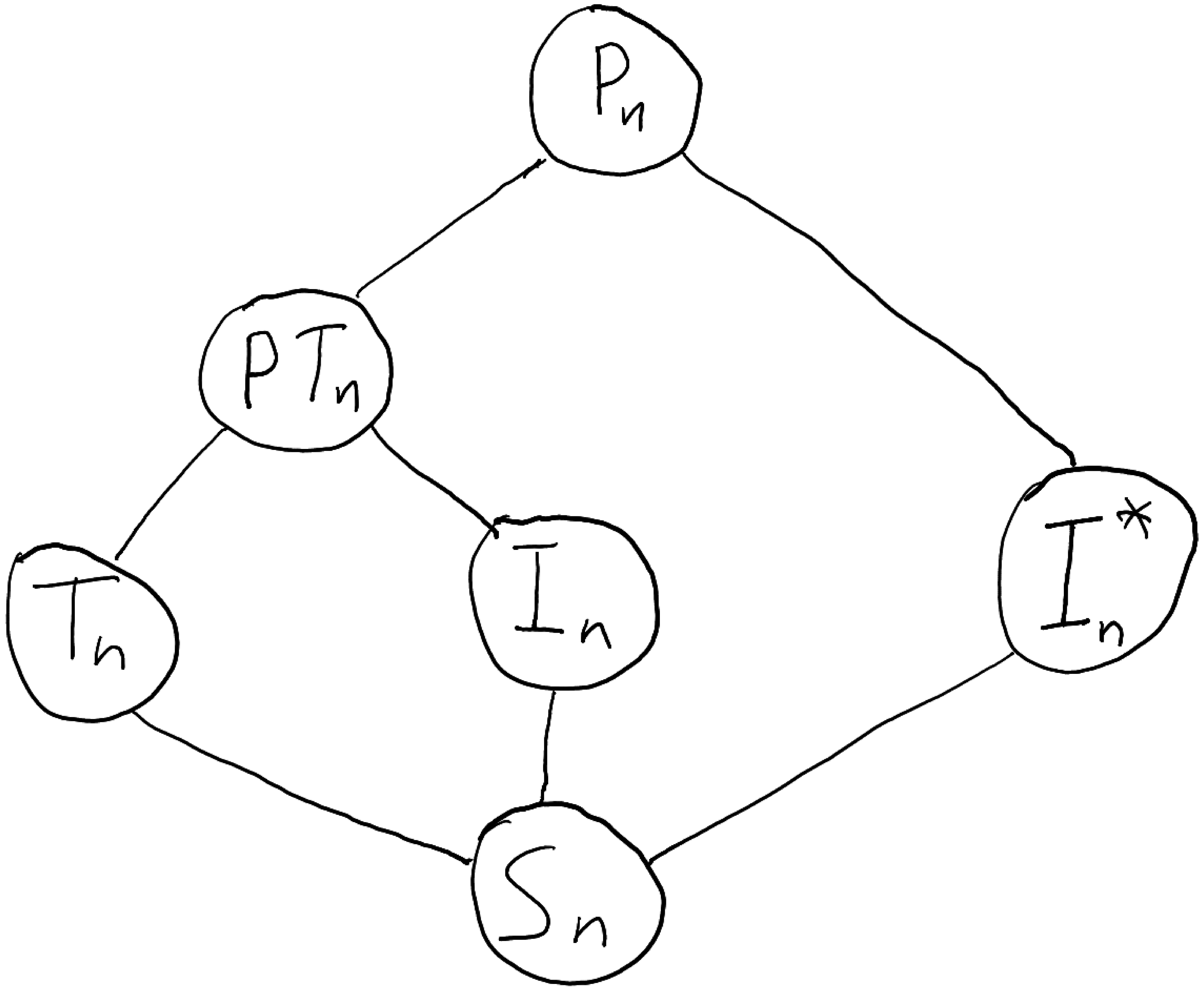
$\in P_3$

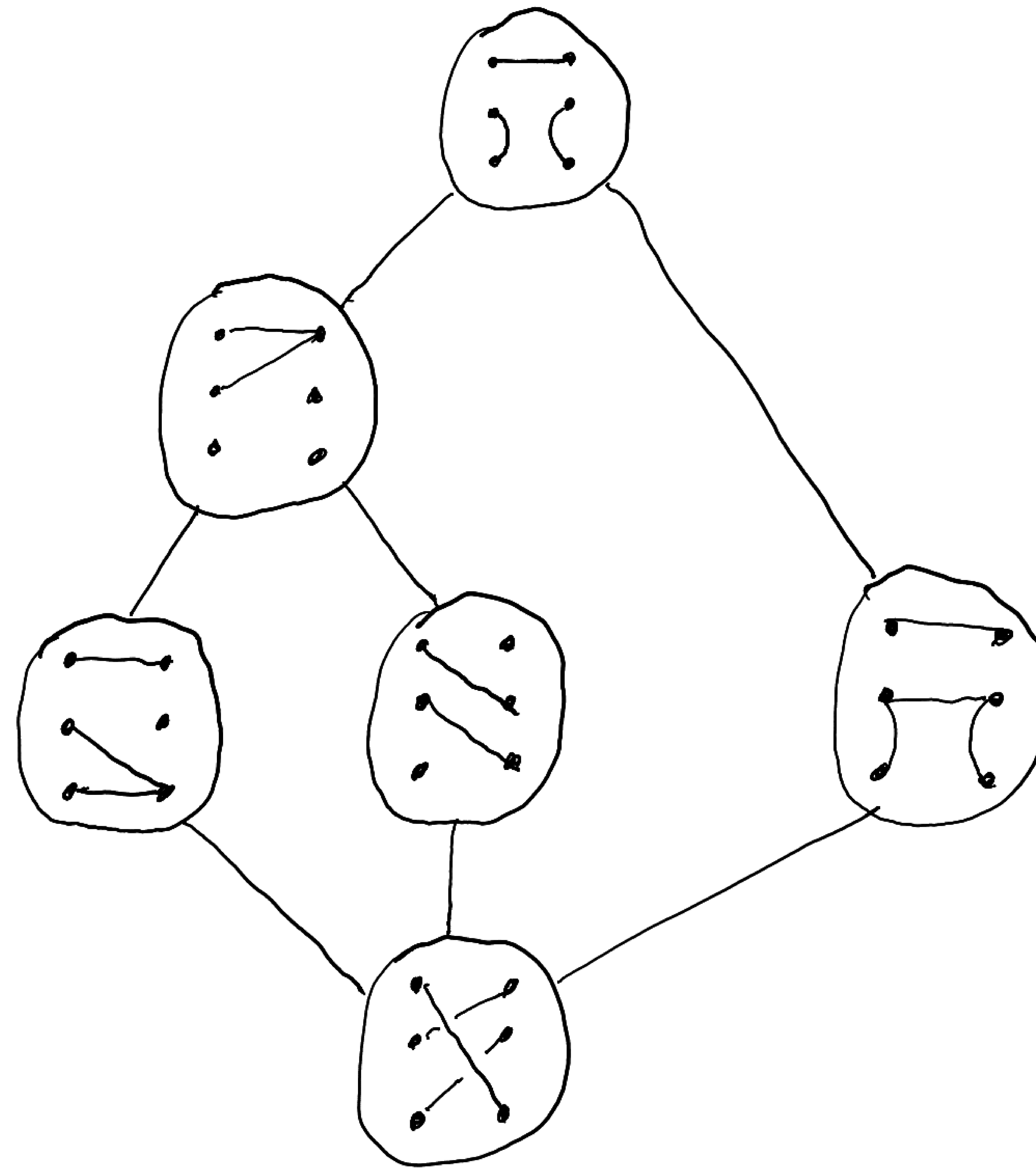
$\in P_2$

Multiplying Partitions

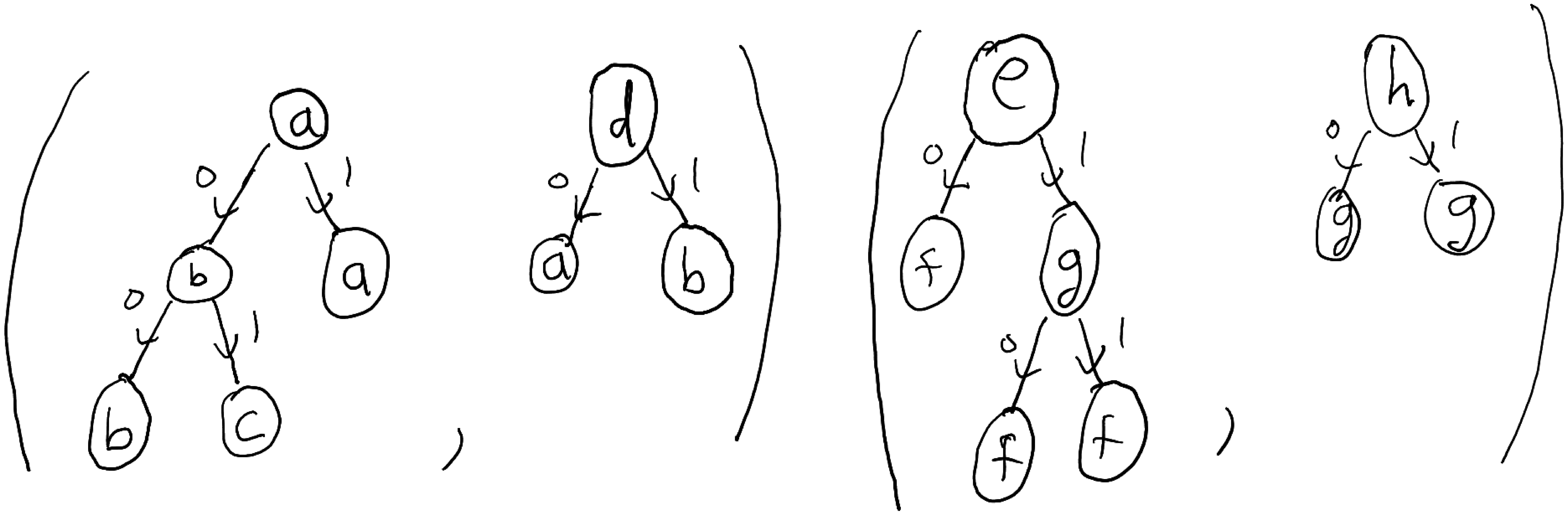




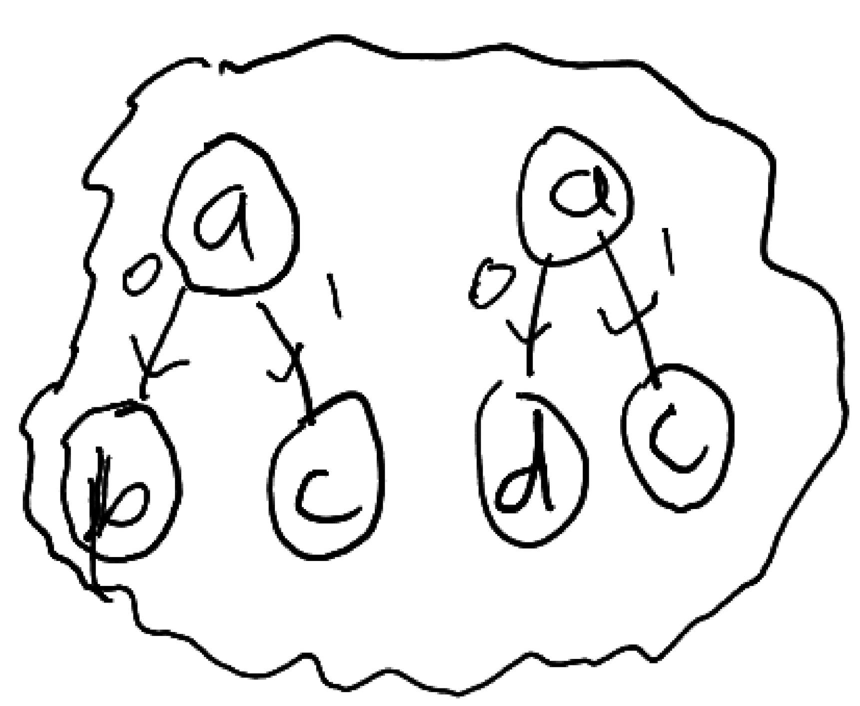

P_n
PT_n
T_n
I_n
I_n^*
S_n



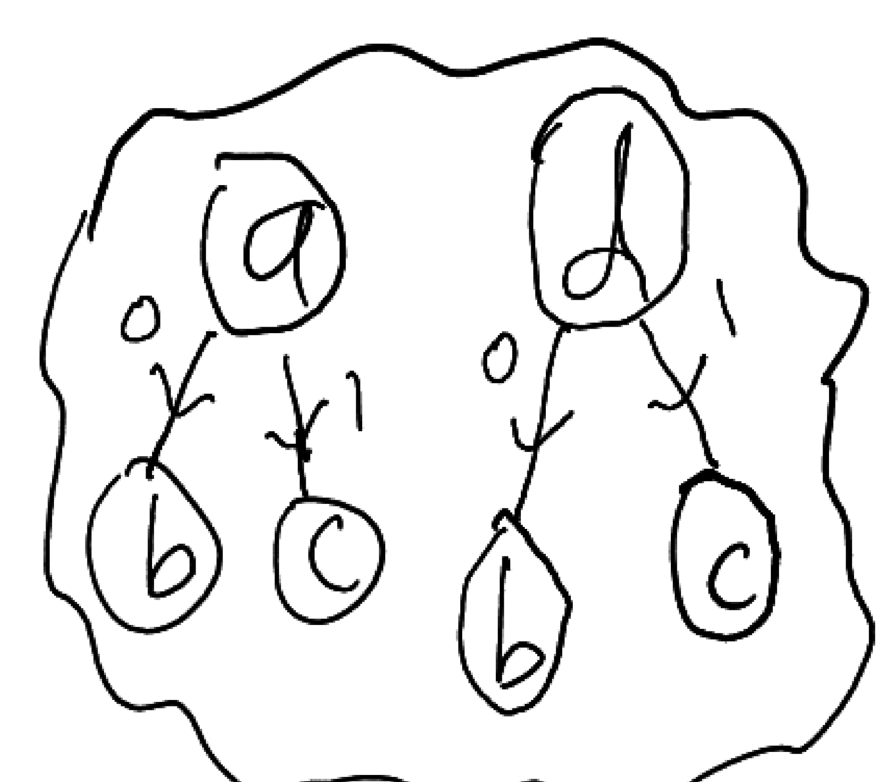
Tree Pair Partitions



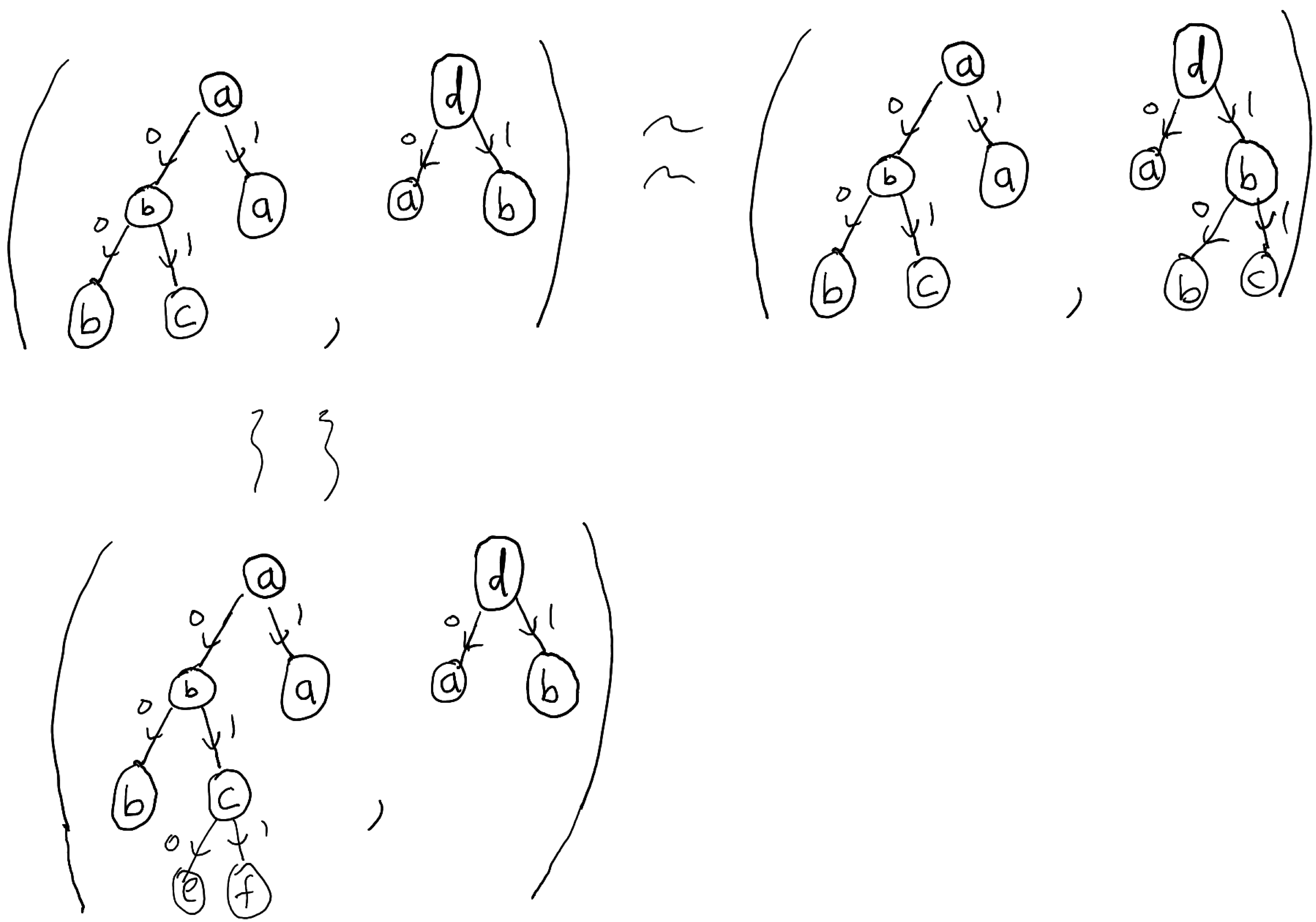
not allowed



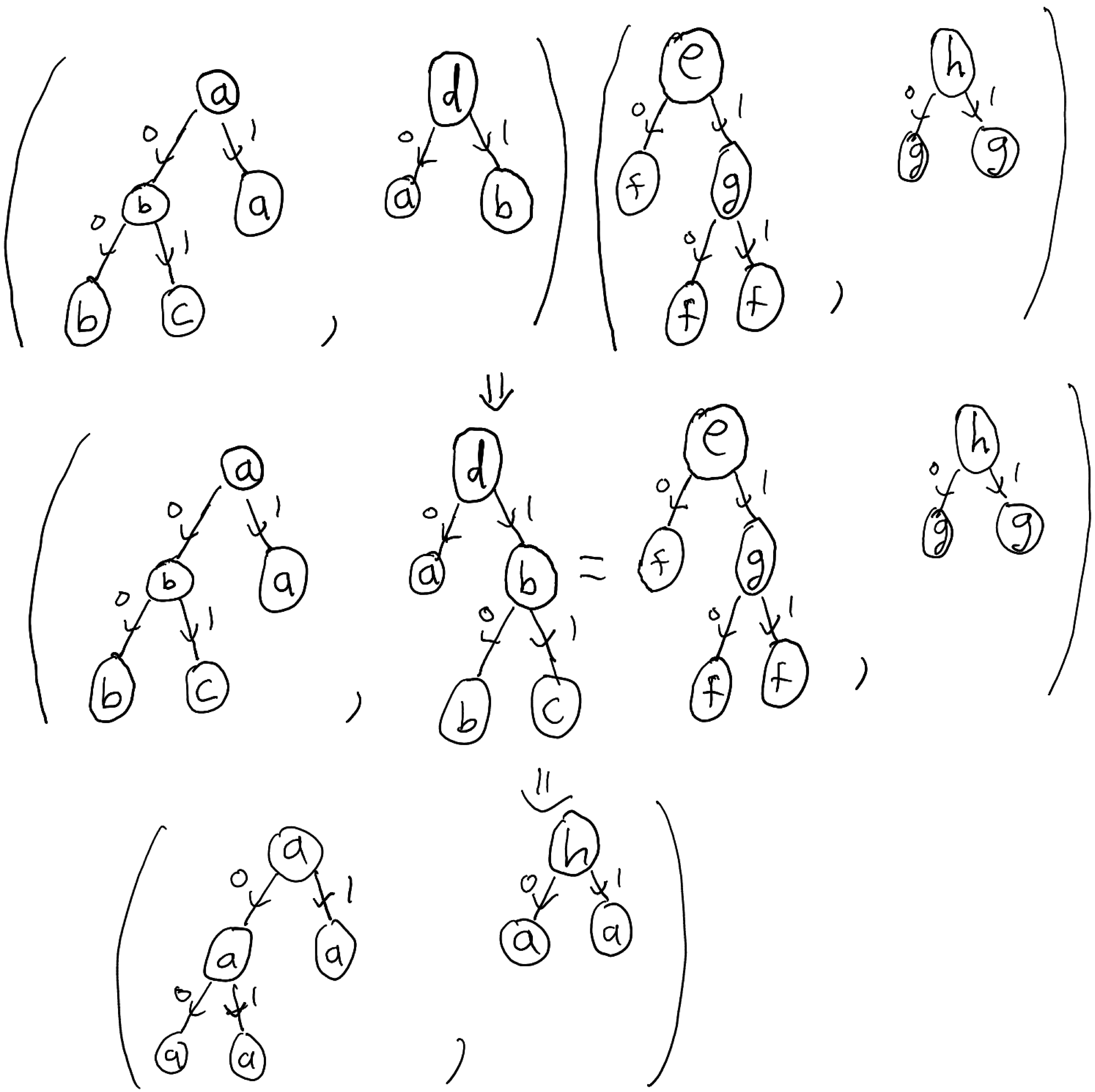
or



Tree Pair Partition Expansions



Example Composition



$d = e$
$a = f$
$b = g$
$b = f$
$c = f$

$\Rightarrow$ $d = e$
$a = b = c = f = g$

Machine Representation

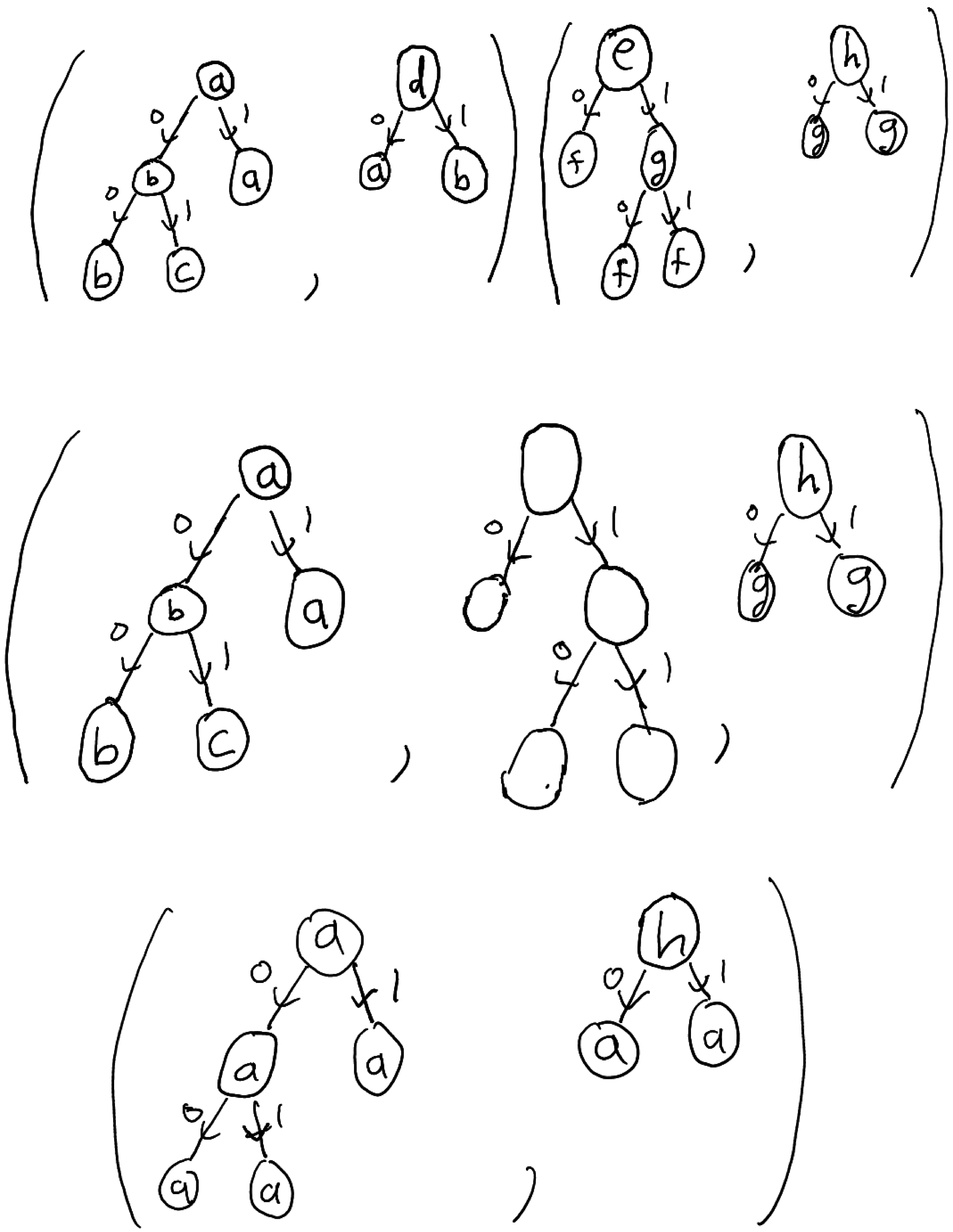
equivalence class of tree pair partions

=

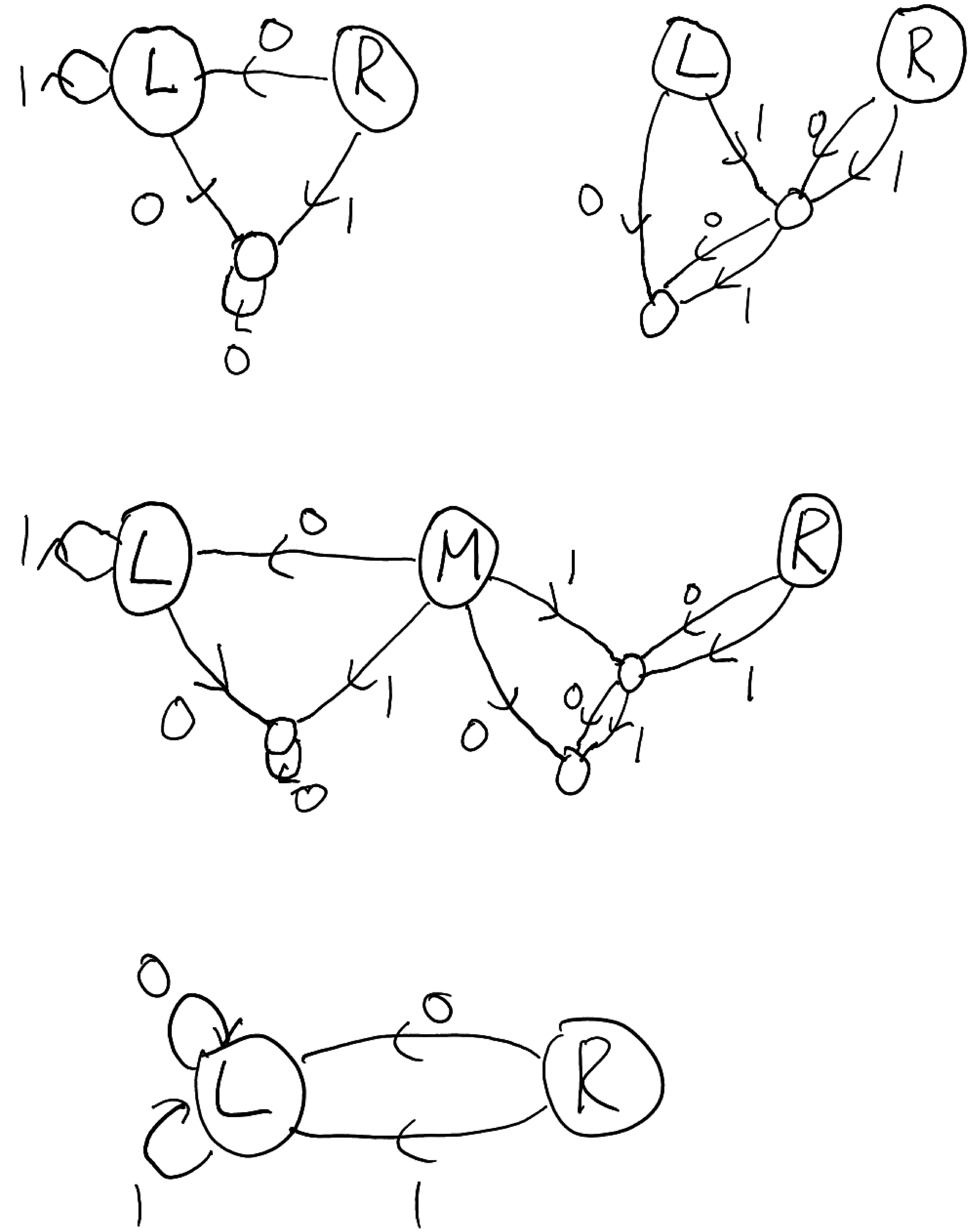
trimmed deterministic transition-collapsed FSA with 2 start states.

Trees to Machines

Trees



Machines



Jónsson - Tarski Stuff

<u>JT algebra</u> $(J, \lambda, \alpha_0, \alpha_1)$

$\lambda : J^2 \to J$
$\alpha_0 : J \to J$
$\alpha_1 : J \to J$

$\alpha : J \to J^2$
is inverse of λ

Variety

<u>JT unary algebra</u> (J, α_0, α_1)

$\alpha_0 : J \to J$
$\alpha_1 : J \to J$

$\alpha : J \to J^2$
is injective

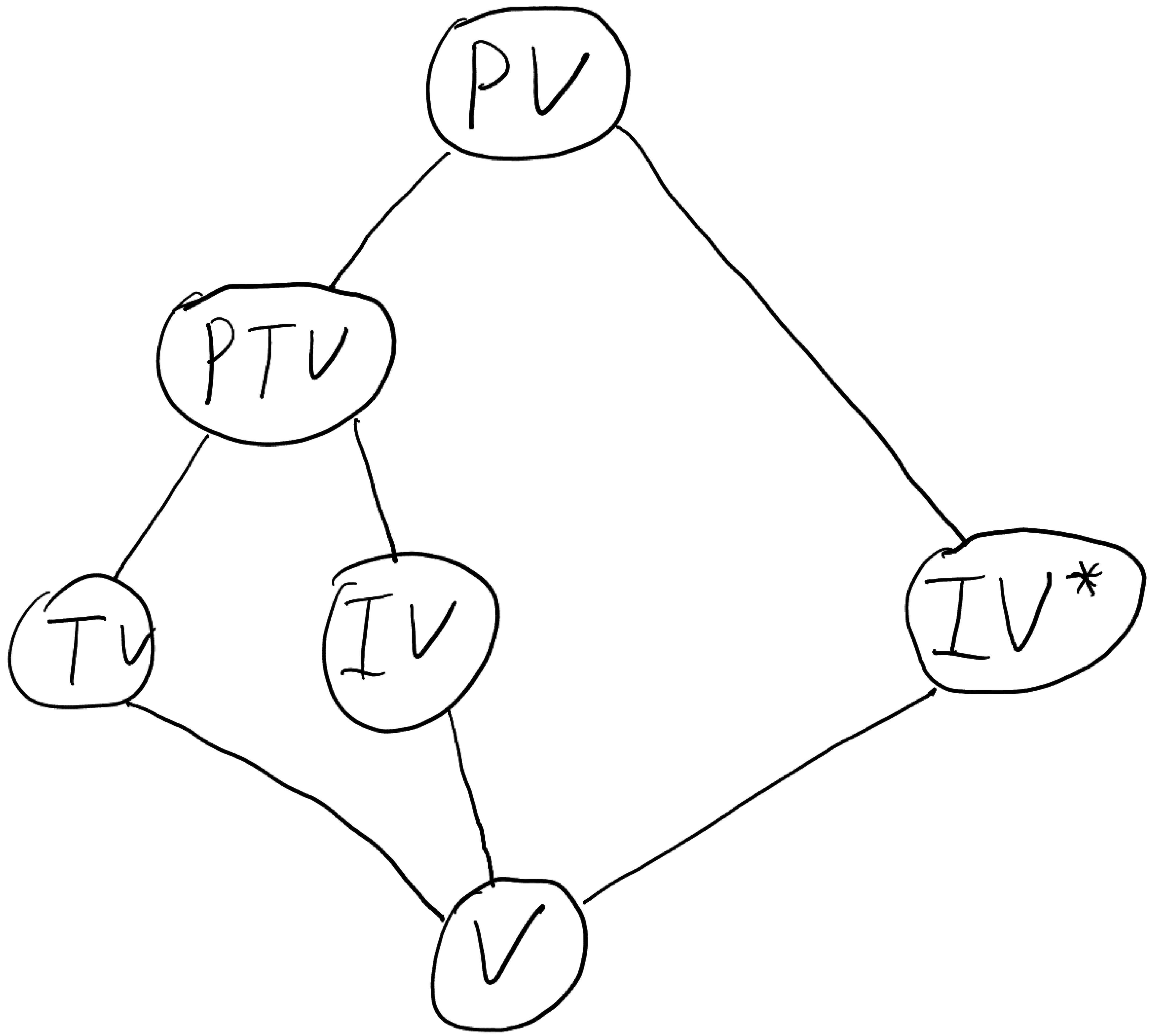
Quasi-Variety

$$
\begin{aligned}
\text{F.p JT algebra} &\approx \text{F.p JT unary algebra} \\
&\approx \text{Binary tree partition} \\
&\approx \text{trimmed transition-collapsed deterministic FSA}
\end{aligned}
$$

V = automorphism group of free JT algebra

P_n = monoid of bijections between subsets of quotients of $\{1, 2, \ldots, n\}$

PV = monoid of isomorphisms between f.g subalgebras of f.p JT algebras


PV
PTV
TV
IV
IV*
V

Theorems

- PV is finitely generated
- PV is not congruence-free

decidable

- $\mathcal{R}$ - same domain JT algebra and same domain f.g subalgebra
- $\mathcal{L}$ - same codomain JT algebra and same codomain f.g subalgebra
- $\mathcal{D}$ - isomorphic f.g subalgebras
- $\mathcal{J}$ - f.g subalgebras embed in quotients of each other

More Theorems (JT)

- f.p JT algebras are coherent
- f.p JT algebras have solvable word problem
- f.p JT algebras have solvable subalgebra membership problem
- f.p JT algebras have solvable isomorphism problem

Questions

1) is PV finitely presented?

2) is IV^* finitely generated?

3) what are the automorphism groups of the other f.p JT algebras (group $\mathcal{H}$-classes of IV^*)?

4) Can PV be represented with the cantor space (probably not)

5) which automorphisms of V extend to PV? (no cantor set connection to use)

6) Planar Partion monoids vs F?

7) other good triples?

8) congruence-free JT algebras give maximal submonoids?
(probably nuclear type-systems work?)

9) is PV (universal?) coCF?

Good Triple conditions

(A, C_{cong}, C_{sub})

- C_{cong} joins
- C_{sub} elt over C_{cong} elt is C_{sub}
- technical mess about extending congruences which agree where defined
- C_{sub} meets
- C_{sub} iso
- equality